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# Vibration analysis of shell-and-tube heat exchangers: an overview—Part 2: vibration response, fretting-wear, guidelines

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**Abstract**

Design guidelines were developed to prevent tube failures due to excessive flow-induced vibration in shell-and-tube heat exchangers. An overview of vibration analysis procedures and recommended design guidelines is presented in this paper. This paper pertains to liquid, gas and two-phase heat exchangers such as nuclear steam generators, reboilers, coolers, service water heat exchangers, condensers, and moisture-separator-reheaters. Part 2 of this paper covers forced vibration excitation mechanisms, vibration response prediction, resulting damage assessment, and acceptance criteria.


## 1. Introduction

Tube failures due to excessive vibration must be avoided in heat exchangers and nuclear steam generators, preferably at the design stage. Thus, a comprehensive flow-induced vibration analysis is required before fabrication of shell-and-tube heat exchangers. It must be shown that tube vibration levels are below allowable levels and that unacceptable resonances and fluidelastic instabilities are avoided.

The purpose of this overview paper is to summarize our design guidelines for flow-induced vibration of heat exchangers. A heat exchanger vibration analysis consists of the following steps: (i) flow distribution calculations, (ii) dynamic parameter evaluation (i.e., damping, effective tube mass, and dynamic stiffness), (iii) formulation of vibration excitation mechanisms, (iv) vibration response prediction, and (v) resulting damage assessment (i.e., comparison against allowables). The requirements applicable to each step are outlined in this paper. It is divided in two parts: Part 1 (Pettigrew and Taylor, 2003) covers flow calculations, dynamic parameters and fluidelastic instability, and Part 2 covers forced vibration excitation mechanisms, vibration response prediction, fretting-wear damage assessment, and acceptance criteria.

## 2. Forced vibration excitation mechanisms

### 2.1. Random excitation

Flow turbulence is a significant excitation mechanism in both liquid and two-phase cross flow. To be able to compare the data and find an upper bound, the excitation forces per unit length must be presented as a normalized excitation

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force spectra. Researchers in this field such as Taylor and Pettigrew (2000), and Pettigrew and Gorman (1981) have used various methods of normalizing their results. Therefore, it was necessary to select one means of normalization and apply it to all of the data. The adopted method is the "equivalent power spectral density (EPSD)", first described by Axisa et al. (1990).

The power spectral density, $\tilde{S}_F(f)$, can be rendered dimensionless using a pressure scaling factor, $p_0$, and a frequency scaling factor, $f_0$, as follows:

$$\tilde{S}_F(f_R) = \frac{S_F(f)}{(p_0 D)^2} f_0, \tag{1}$$

where $f_R$ is the reduced frequency, defined as $f/f_0$, and $D$ is the tube diameter.

A difficulty arises in the calculation of $S_F(f)$ because the correlation length, $\lambda_c$, is rarely known. Axisa et al. (1990) present an EPSD, $\tilde{S}_F(f_R)e$, defined as follows:

$$\tilde{S}_F(f_R)_e = \frac{\lambda_c}{L_e} \tilde{S}_F(f_R), \tag{2}$$

where $L_e$ is the excited tube length. Using this definition, the EPSD for mode 1 can be defined in terms of tube displacement, $y(x)$, as follows:

$$\tilde{S}_F(f_R)_e = \frac{\overline{y^2(x)_1} 64\pi^3 f_1^3 m^2 \zeta_1}{\phi_1^2 a_1} \frac{1}{(p_0 D)^2} f_0, \tag{3}$$

where $\phi_1(x)$ is the normalized mode shape for the 1st mode, $a_1$ is the 1st mode dimensionless coefficient for the modal joint acceptance, $f_1$ is the 1st mode tube natural frequency, $m$ is the total tube mass (tube mass + hydrodynamic mass) and $\zeta_1$ is the damping ratio for the 1st mode. Values of $\phi_1^2(x_{\max})$and $a_1$ for a variety of end conditions are given in Table 1. Values for the modal factor, $a_1$, as given in Table 1 were first published by Antunes (1986) and then later included in a journal paper by Axisa et al. (1990). There are several assumptions made in deriving Eq. (3): a single dominant mode with a small damping ratio exists, the modes are reasonably separated and the correlation length is much less than unity.

Using Eq. (3), the mean square of tube displacement can be found without knowledge of the correlation length. Instead, a small correlation length has been assumed. To correctly compare spectra obtained using experimental rigs with varying geometries, it is necessary to define a reference EPSD, $\tilde{S}_F(f_R)_e^0$, based on a reference excited tube length, $L_0$:

$$\tilde{S}_F(f_R)_e^0 = \tilde{S}_F(f_R)_e \frac{L_e}{L_0}, \tag{4}$$

where $L_e$ is the excited tube length. In this paper, a reference length of $L_0 = 1\,\text{m}$ is applied.

### 2.1.1. Single phase flow

In liquid flow, two distinct flow fields are possible. Interior tubes, well within a heat exchanger tube bundle, are excited by turbulence generated within the bundle. This excitation is governed by the tube bundle geometry. On the other hand, upstream or inlet tubes are excited by turbulence generated by upstream components such as inlet nozzles, entrance ports and upstream piping elements. Upstream turbulence levels are governed by the upstream flow path geometry and are very often much larger than those generated within the bundle. Such excitation is often referred to as far-field excitation.

Random excitation is usually not a problem with gas or vapour cross flow. The pressure fluctuations and resulting excitation forces due to gas cross flow at a given velocity are generally an order of magnitude less than those for a liquid or two-phase mixture at the same velocity. However, gas velocities can be extremely high and at high pressure the densities can be significant. Therefore, some consideration should be given to random excitation in high-pressure gas heat exchangers.

Table 1
Modal factor ($a_1$) and mode shape ($\phi_1^2(x_{\max})$) constants for mode 1

| | Rigid tube | Clamped–clamped | Clamped–pinned | Clamped–free | Pinned–pinned |
|---|---|---|---|---|---|
| Modal factor | 2 | 0.8 | 0.9 | 0.5 | 1.1 |
| Mode shape | 1.0 Translation | 2.522 | 2.278 | 4.0 | 2.0 |

Taylor and Pettigrew (2000) combined data from many sources to arrive at the reference EPSD guideline shown in Fig. 1. The lower bound, shown in Fig. 1, should be used when the upstream turbulence is less than or equal to the turbulence within the tube bundle. The upper bound, shown in Fig. 1, should be used if the upstream turbulence exceeds the turbulence inside the tube bundle. The shape of the upper bound was assumed to be similar to lower bound. The number of upper bound (inlet flow) data points was too limited to clearly indicate a different shape. The boundaries are defined as follows:

$$\begin{aligned}\text{Interior}\quad \tilde{S}_F(f_R)^0_e &= 4\times10^{-4}(f/f_0)^{-0.5}, \quad 0.01<f/f_0<0.5,\\ \tilde{S}_F(f_R)^0_e &= 5\times10^{-5}(f/f_0)^{-3.5}, \quad 0.5<f/f_0,\end{aligned} \tag{5}$$

$$\begin{aligned}\text{Inlet}\quad \tilde{S}_F(f_R)^0_e &= 1\times10^{-2}(f/f_0)^{-0.5}, \quad 0.01<f/f_0<0.5,\\ \tilde{S}_F(f_R)^0_e &= 1.25\times10^{-3}(f/f_0)^{-3.5}, \quad 0.5<f/f_0.\end{aligned} \tag{6}$$

For single-phase flow, $f_0 = U_p/D$ and $p_0 = \rho U_p^2/2$.

In most cases, the random excitation forces for interior tubes are significantly lower than for upstream tubes. The vibration response of the upstream tubes will be larger. Thus, it may not be necessary to consider the vibration response of interior tubes when they are otherwise identical to the upstream tubes.

The above formulation (Eq. (6)) should also apply to upstream finned tubes. Because upstream turbulence is dominant, the presence of fins should not significantly affect turbulence levels.

### 2.1.2. Two-phase flow

Random turbulence excitation forces can be significant in two-phase cross flow, in particular, in the U-bend region of steam generators. The term turbulence is used loosely in two-phase flows. It describes the dynamics of the two-phase mixture as it flows through a tube bundle.

For void fractions of 10% or less, two-phase flow random forces behave like single-phase flow forces and the single-phase guidelines can be used. At higher void fractions, the effect of the two-phase mixture begins to dominate the single-phase random forces. The physics of these two-phase forces are not well understood but recent study of the effect of void fraction and flow regime have led to reasonable collapse of two-phase data from air–water, Freon liquid–vapour, Freon vapour–water and steam–water experiments.

Preliminary two-phase design guidelines for random excitation of heat exchanger tubes have been presented by several authors such as Taylor et al. (1989), Axisa et al. (1990) and Taylor et al. (1996). Most of these data were obtained with air–water mixtures with some steam–water data. More recently, de Langre and Villard (1998) introduced a dimensionless scaling that simplifies that suggested by Taylor et al. (1996) and more closely follows the principle used in the scaling of single-phase random forces.

De Langre and Villard (1998) show that a two-phase power spectral density, $\tilde{S}_F(f)$, can be rendered dimensionless using Eq. (1) with a two-phase pressure scaling factor, $p_0 = \rho_{TP} g D_w$, and a two-phase frequency scaling factor, $f_0 = U_p/D_w$. The length scale, $D_w$, is defined as follows:

$$D_w = 0.1D/\sqrt{1-\varepsilon_g}. \tag{7}$$

The selection of these scaling factors is described is some detail in (de Langre and Villard, 1998). Many pressure scaling factors were considered, but the gravity based factor was the most efficient. The authors point out that gravity in the scaling factor may be related to a dynamic pressure built out of the drift velocities between the gas and liquid phases. The length scale, $D_w$, is a simplified version of the characteristic void length introduced by Taylor et al. (1996). This length scale recognizes the effect of the gas–liquid patterns on the frequency characteristics of the two-phase random forces.

Using these scaling factors and the reference EPSD described above in Eq. (4), data from many sources were collapsed on a single plot (see Fig. 2). These are the same data that are plotted in de Langre and Villard (1998). The following boundary spectra for two-phase random forces are similar to the de Langre and Villard boundary, but not identical, since the reference diameter ratio they introduced has not been included in Eq. (4):

$$\begin{aligned}\tilde{S}_F(f_R)^0_e &= 16(f/f_0)^{-0.5}, \quad 0.001\leqslant f/f_0\leqslant0.05,\\ \tilde{S}_F(f_R)^0_e &= 2\times10^{-3}(f/f_0)^{-3.5}, \quad 0.05\leqslant f/f_0\leqslant1.\end{aligned} \tag{8}$$

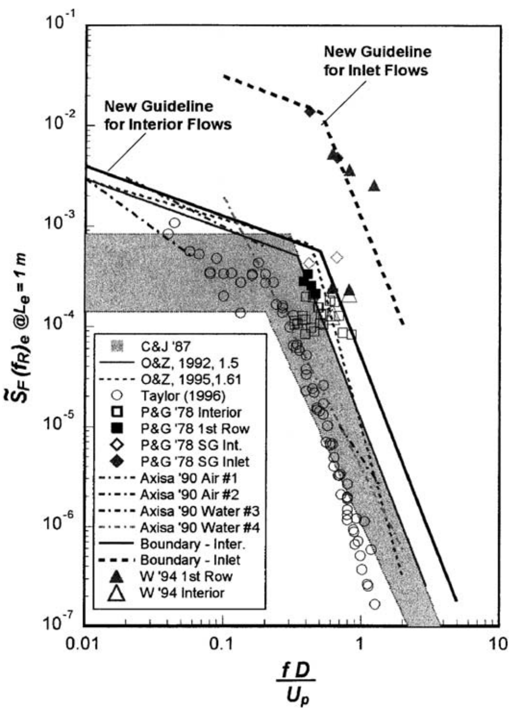


Fig. 1. Proposed guideline for single-phase random excitation forces (references provided in the legend can be found in Taylor and Pettigrew (2000)).

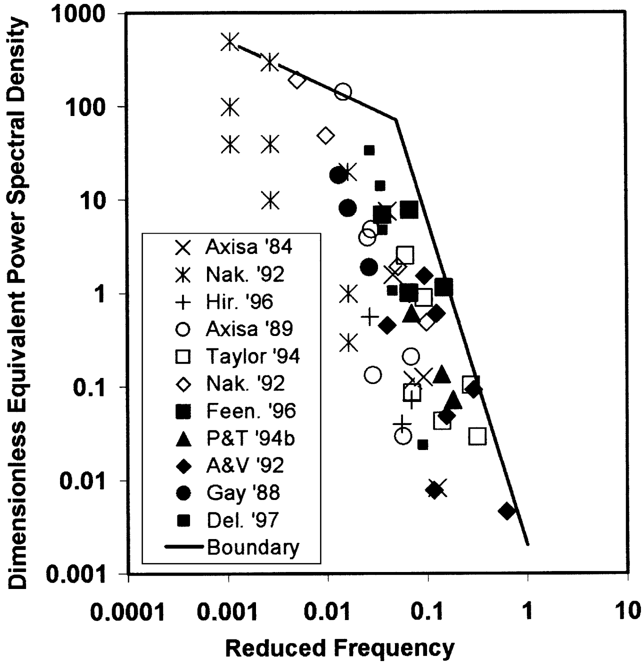


Fig. 2. Proposed guideline for two-phase random excitation forces: line symbols—steam–water tests, empty symbols—air–water tests, and filled symbols—Freon tests (references provided in the legend can be found in de Langre and Villard (1998)).

### 2.2. Periodic wake shedding

Periodic wake shedding, or vortex shedding, may be a problem when the shedding frequency coincides with a tube natural frequency. This may lead to resonance and large vibration amplitudes. Periodic wake shedding resonance has been observed by Pettigrew and Gorman (1981) in tube bundles subjected to liquid cross flow.

Periodic wake shedding resonance may be of concern in liquid cross flow where the flow is relatively uniform. It is not normally a problem at the entrance region of steam generators because the flow is very nonuniform and quite turbulent (Pettigrew et al., 1973). Turbulence inhibits periodic wake shedding (Cheung and Melbourne, 1983). Periodic wake shedding is generally not a problem in two-phase flow except at very low void fractions (i.e., $\varepsilon_g < 15\%$), Pettigrew and Taylor (1994). Then, the behaviour is similar to liquid flow.

Periodic wake shedding resonance is usually not a problem in gas heat exchangers. The gas density is usually too low to cause significant periodic forces at flow velocities close to resonance. Normal flow velocities in gas heat exchangers are usually much higher than those required for resonance. However, it may be possible in high-pressure components such as MSRs with higher density gas on the shell side. It could also happen for higher modes of vibration with higher frequencies corresponding to higher flow velocities. Thus, periodic wake shedding resonance cannot always be ignored in gas heat exchangers.

Generally, there is little information on the magnitude of periodic wake shedding forces in tube bundles. There is more information on periodic wake shedding frequencies. However, this information is often contradictory.

Periodic wake shedding is described in terms of a Strouhal number $S = f_s D/U_p$, which formulates the wake shedding frequency, $f_s$, and a fluctuating lift or drag coefficient, $C_L$, which is used to estimate the periodic forces, $F_s$, due to wake shedding. Thus,

$$F_s = C_L D \rho U_p^2/2 \tag{9}$$

Mair et al. (1975) show that it is appropriate to use the equivalent hydraulic diameter, $D_h$, to estimate Strouhal numbers and fluid forces for finned tubes. Thus $D = D_h$ and $P/D = P/D_h$ for finned tubes.

Periodic wake shedding data were reviewed for tube bundles of various configurations. Fig. 3 shows Strouhal numbers for tube bundles of various configurations and $P/D$ in liquid flow. The Strouhal numbers based on the pitch velocity are generally within 0.33 and 0.67 for heat exchanger tube bundles of $P/D$ between 1.23 and 1.57 (Pettigrew and Gorman, 1981). Weaver et al. (1987) and Weaver (1993) also reviewed the question of wake shedding in tube bundles. These data are summarized in Fig. 4 where the Strouhal numbers, $S_u$, are defined in terms of the approach velocity, $U_\infty$. Although there is a lot of scatter in the data, expressions based on the theory of Owen (1965) are proposed to formulate the average values as shown by the curves in Fig. 4. These expressions can easily be transformed to yield Strouhal numbers, S, defined in terms of the pitch velocity. Thus,

$$S = (1/1.73)(D/P) \tag{10a}$$

for normal triangular bundles;

$$S = (1/1.16)(D/P) \tag{10b}$$

for rotated triangular bundles; and

$$S = (1/2)(D/P) \tag{10c}$$

for both normal and rotated square bundles. These expressions give Strouhal numbers between 0.32 and 0.70 for realistic heat exchanger tube bundles of $P/D$ between 1.23 and 1.57.

These numbers are very similar to those found in liquids as shown in Fig. 3. These correspond to dimensionless velocities $U_p/fD$ between 1.5 and 3.0. Resonance should be assumed possible within this velocity range. In the case of a tube subjected to a non-uniform velocity, only the fluctuating fluid forces corresponding to the region within the above range of dimensionless velocities need to be considered.

A review of the available data showed that fluctuating forces due to periodic wake shedding depend on several parameters such as bundle configuration, location within the bundle, Reynolds number, turbulence level, fluid density and $P/D$. At the limit when $P/D$ is large, fluctuating force coefficients should approach those for single cylinders. On the other hand, when $P/D$ is very small the force coefficients are small since the fluid mass associated with the formation of periodic wake or vortices should be small as there is little space available within the bundle for large vortices.

The available data for fluctuating force coefficients is compiled in Fig. 5. It shows that the fluctuating lift coefficient, $C_L$, is very dependent on $P/D$ up to $P/D \approx 2.5$. For heat exchanger tube bundles of $P/D < 1.6$, a fluctuating

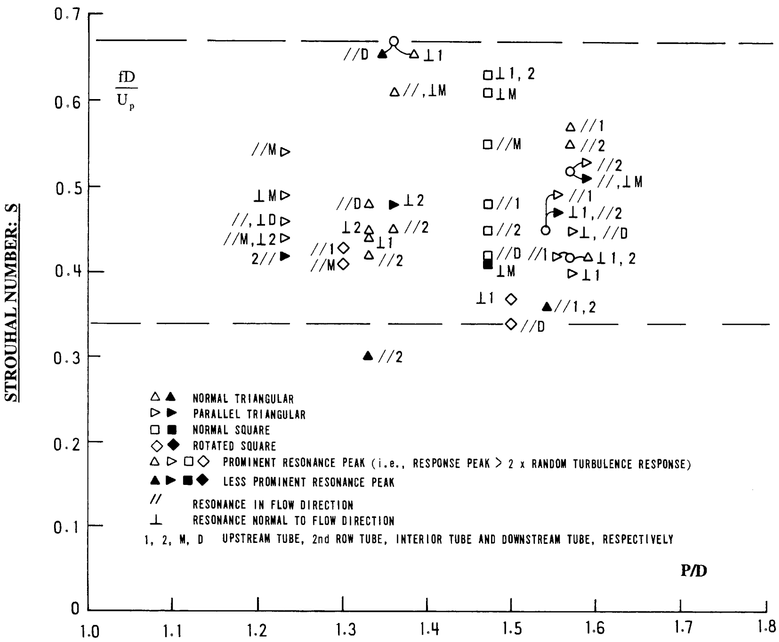


Fig. 3. Strouhal numbers for tube bundles in liquid cross flow, based on pitch velocity (Pettigrew and Gorman, 1981).

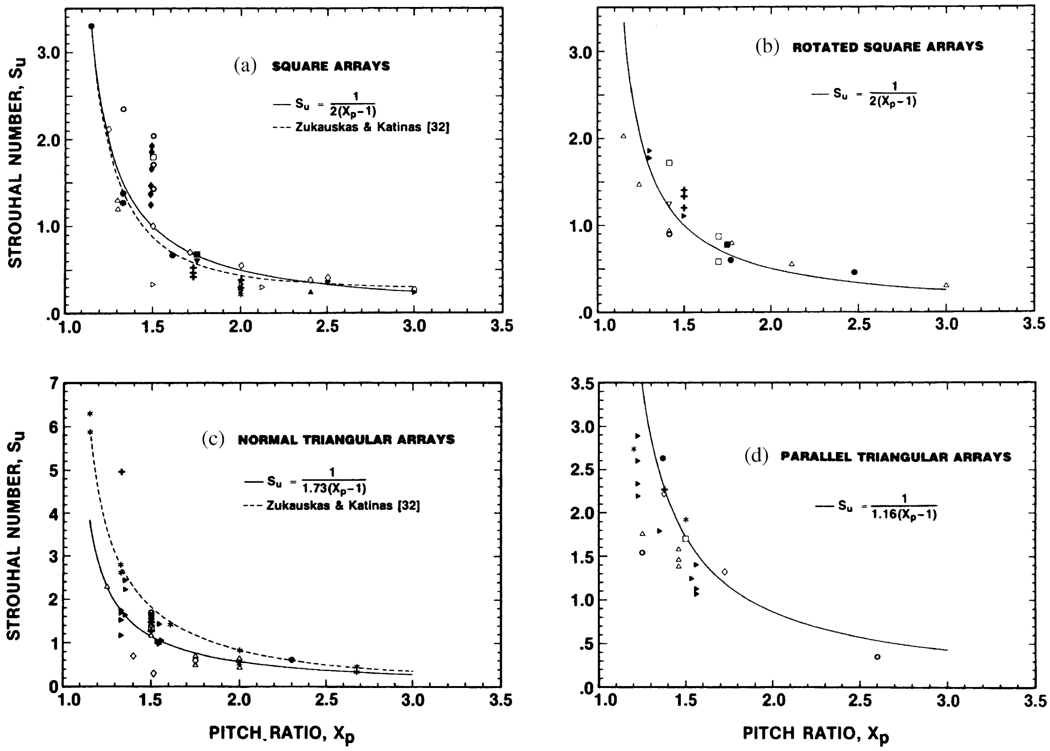


Fig. 4. Summary of periodic wake shedding data in tube bundle: Strouhal number, $S_u$, based on approach velocity (Weaver et al., 1987).

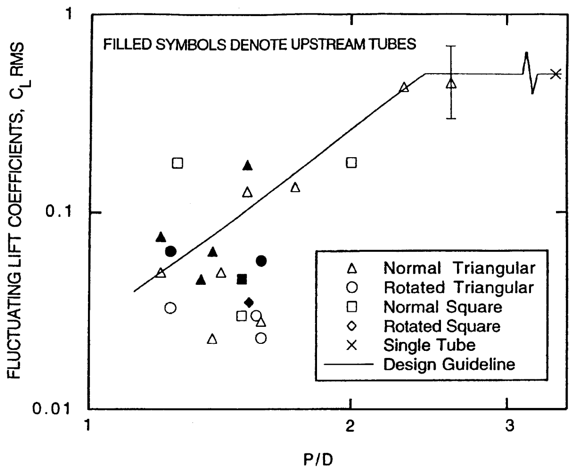


Fig. 5. Fluctuating force coefficients for tube bundles in cross flow.

force coefficient:

$$C_L = 0.075\text{r.m.s.}$$

is recommended to calculate periodic wake shedding forces.

When resonance is considered possible, the maximum allowable tube vibration amplitude should not exceed $0.02D$ or 2% of the tube diameter. Below $0.02D$, the vibration amplitude is generally not sufficient to correlate periodic wake shedding along the tube, resulting in much lower vibration response.

### 2.3. Acoustic resonance

Acoustic resonance may take place in heat exchanger tube bundles when vortex or periodic wake shedding frequencies coincide with a natural frequency for acoustic standing waves within a heat exchanger. Such resonance normally causes intense acoustic noise and often serious tube and baffle damage. Acoustic resonance is possible in gas heat exchangers with both finned and unfinned tubes.

Acoustic resonance requires two conditions: (i) coincidence of shedding and acoustic frequency, and (ii) sufficient acoustic energy or sufficiently low acoustic damping to allow sustained acoustic standing wave resonance. In heat exchanger tube bundles, acoustic standing waves are generally normal to both the tube axes and the flow direction.

#### 2.3.1. Frequency estimates

As discussed earlier, periodic wake shedding is governed by the Strouhal number:

$$S = f_s D / U_p. \tag{11}$$

The Strouhal number may be obtained from Eq. (10) or from Figs. 3 and 4 for the different tube bundle configurations and $P/D$ ratios. The range of Strouhal numbers over which acoustic resonance can take place should be extended to allow for possible "lock–in" of the wake. As suggested by Blevins and Bressler (1987), acoustic resonance may be possible between 0.8 and 1.3 S or in terms of vortex shedding frequency,

$$0.8SU_p/D < f_s < 1.35SU_p/D. \tag{12}$$

Acoustic standing wave frequencies, $f_{an}$, are defined by

$$f_{an} = nC/2W, \tag{13}$$

where $W$ is the dimension of the heat exchanger tube bundle cavity in the direction normal to the flow and the tube axes, $n$ is the mode order and $C$ is the effective speed of sound (Ziada et al., 1989)

$$C \approx C_0/\sqrt{1+\sigma}. \tag{14}$$

The speed of sound $C_0$ is obtained from

$$C_0 = \sqrt{kp/\rho}, \tag{15}$$

where $k$ is the specific heat ratio ($k = 1.33$ for steam), $p$ is the shell side pressure and $\rho$ the shell side fluid density.

The speed of sound is affected by the presence of the tubes. This effect is related to the solidity ratio, $\sigma$, which is the ratio of the volume occupied by the tubes over the volume of the tube bundle. For a triangular tube bundle

$$\sigma = \pi D^2/(2\sqrt{3}\,P^2) \tag{16a}$$

and for a square tube bundle

$$\sigma = \pi D^2/P^2. \tag{16b}$$

The acoustic standing wave frequencies should be calculated for the first few acoustic modes (i.e., the first five modes should suffice). If one or more acoustic mode frequencies fall within the range of periodic wake shedding frequency, acoustic resonance conditions are possible.

### 2.3.2. Susceptibility of resonance

Coincidence of acoustic and shedding frequencies does not necessarily cause resonance. Resonance also depends on acoustic energy and acoustic damping. Several design criteria have been proposed to evaluate susceptibility to tube bundle resonance. The design guidelines of Blevins and Bressler (1987) and of Ziada et al. (1989) are suggested for heat exchanger tube bundles.

Blevins and Bressler (1987) suggest that resonance is less likely for closely packed tube bundles probably because acoustic damping is higher. Also closely packed tube bundles probably prevent the formation of larger vortices that are associated with higher acoustic energy. From experimental data for first mode acoustic resonance, they show that resonance is unlikely for $P/D < 1.6$ and $L/D < 3.0$ for staggered (triangular) tube bundles, and for $L/D < 1.4$ for in-line (square) tube bundles. Here $P$ is the transverse pitch and $L$ is the longitudinal pitch. However, they do not discuss the applicability of their criterion to higher order acoustic modes.

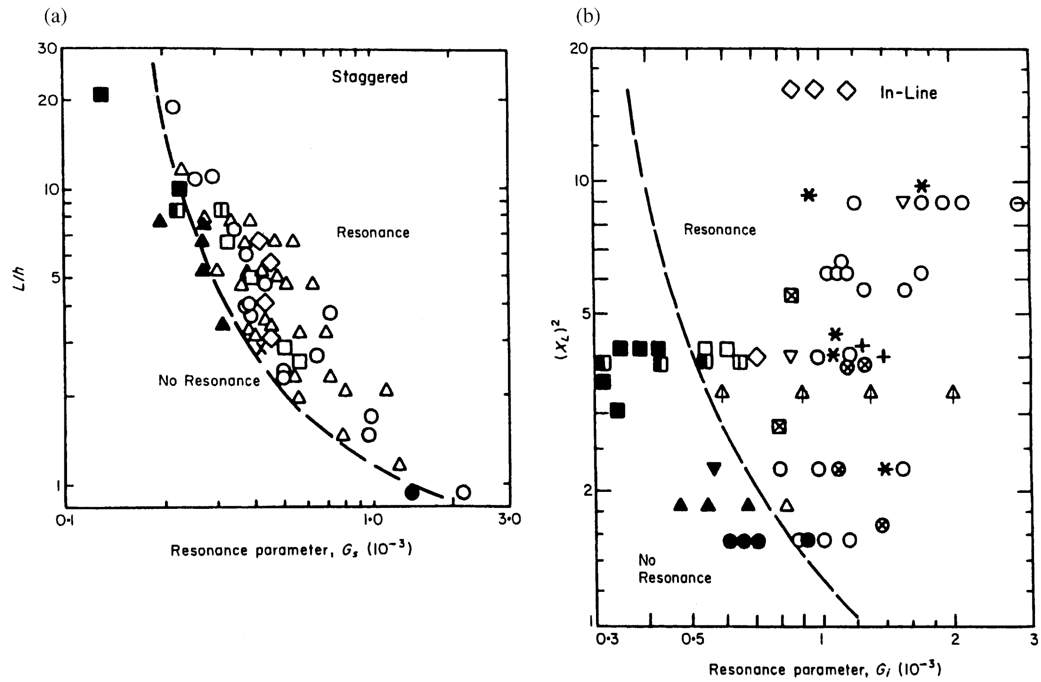


Fig. 6. Proposed resonance parameter for: (a) Staggered (triangular) arrays; in-line (square) (b) arrays (Ziada et al., 1989).

Ziada et al. (1989) propose a resonance parameter to assess susceptibility to resonance. For staggered arrays (triangular), this parameter has the form

$$G_s = \sqrt{R_{cr}}\left[\frac{\sqrt{(L/D)(P/D-1)}}{(L/D)-1}\right]\left(\frac{v}{CD}\right), \tag{17a}$$

where $R_{cr}$ is the Reynolds number based on the critical flow velocity at resonance and $v$ is the kinematic viscosity for the shell side gas. The resonance parameter, $G_s$, is correlated against the flow path parameter, $L/h$, in Fig. 6a for both resonance and non-resonance data. The parameter $L/h$, defined in (Ziada et al., 1989), represents the ratio between the length, $L$, of the flow jet emerging between the tubes and the minimum flow gap or thickness, $h$, of this jet. It is a measure of the stability of the flow jet and hence the propensity to resonance.

Similarly, for in-line arrays (square)

$$G_i = \sqrt{R_{cr}}(P/D)\left(\frac{v}{CD}\right). \tag{17b}$$

The parameter $G_i$ is plotted against $L/D$ in Fig. 6b. For a satisfactory design it must be shown that the heat exchanger tube bundle is in the non-resonance part of Fig. 6.

## 3. Vibration response prediction

Due to the complexity of a multispan heat exchanger tube, a computer code must be used to predict vibration response accurately and to determine the susceptibility of a tube to periodic wake shedding resonance and fluidelastic instability. This computer code must be capable of calculating the mode shapes and natural frequencies for a number of modes at least equal to the number of tube spans.

Generally the code will be in two parts: the first part will calculate the free vibration tube characteristics such as mode shapes and natural frequencies, while the second part is a forced vibration analysis that calculates the tube response to turbulence-induced excitation and periodic wake shedding, and the fluidelastic instability ratios. A forced vibration analysis involves the application of the distributed flow velocities and densities that were determined using the methods outlined in (Pettigrew and Taylor, 2003).

From a mechanics point-of-view, the tubes are simply multispan beams clamped at the tubesheet and held at the supports with varying degrees of constraint. To predict tube response, it is convenient and appropriate to assume that the intermediate supports are hinged. With this assumption, the analysis is linear and either a finite-element code or an analytical code can be used.

If the tube-to-support clearances are too large, the tube supports will not be effective and the assumption of hinged supports will not be valid. Tube-to-support diametral clearances of 0.38 mm (0.015 in) or less are typically used in nuclear heat exchanger design. Effective supports can become ineffective if a heat exchanger is subjected to significant corrosion or a chemical cleaning technique is used to clean a fouled unit. The effects of future chemical cleaning and corrosion should be considered in the vibration response analysis.

The ultimate goal of vibration analyses is to ensure that fretting-wear or fatigue damage does not occur in heat exchangers. Although a linear analysis does not predict fretting-wear, significant fretting-wear can be avoided by keeping the predicted vibration amplitudes below the allowables provided later in this paper. In addition, fretting-wear damage can be estimated as discussed in the next section.

Within vibration predicting codes, the tube response $y(x,t)$ to a distributed force $g(x,t)$ at any point $x$ along the tube and at any time $t$ may be expressed as a normal mode expansion in terms of the generalized coordinates $q_i(t)$ as follows:

$$y(x,t) = \sum_{i=1}^{n} \phi_i(x) q_i(t), \tag{18}$$

where $\phi_i(x)$ is the mode shape of the $i$th mode and $n$ is the number of modes to be used in the analysis. Using Lagrange's equation and assuming that the damping is small and that it does not introduce coupling between modes, the equation of motion for the $i$th mode is as follows:

$$\ddot{q}_i(t) + 2\omega_i\zeta_i\dot{q}_i(t) + \omega_i^2 q_i(t) = \int_o^{\ell} g(x,t)\phi_i(x)\,\mathrm{d}x, \tag{19}$$

where $\zeta_i$ is the damping ratio, $m$ is the mass per unit length, $l$ is the tube length and $\omega_i$ is the angular frequency of the $i$th natural mode. The natural modes are normalized so that

$$\int_o^{\ell} m\phi_i^2(x)\,\mathrm{d}x = 1. \tag{20}$$

Knowing $m$, $\zeta$, $i$, the support locations and the flexural rigidity of the tube, the response to different types of forcing functions $g(x, t)$ is obtained by solving the family of equations described in Eqs. (18) and (19). Assumptions that can be used within this type of code include uniform mass distribution and flexural rigidity, negligible effect of shear and rotary inertia, no axial motion of any point along the tube, homogeneous boundary conditions and continuity at the intermediate supports.

### 3.1. Fluidelastic instability

A fluidelastic instability ratio is determined by calculating an effective velocity based on the velocity and density profiles and the important mode shapes. For the $i$th mode, the critical velocity $U_{pci}$, at which instability occurs, may be expressed as follows:

$$\frac{U_{pci}}{fD} = K\left[\frac{2\pi\xi_i}{D^2\rho\int_0^{\ell} u^2(v)r(x)\phi^2(x)\,\mathrm{d}x}\right]^{1/2}, \tag{21}$$

where nonuniform flow velocities and densities are represented as $U_p(x) = U_p u(x)$ and $\rho(x) = \rho r(x)$. This is a generalized form of Eq. (28) in Part 1 of this paper.

### 3.2. Random excitation

Tube response to turbulence-induced excitation forces must be calculated using random vibration theory, and the response must be summed over all significant modes of vibration. The mean square amplitude response, $\overline{y^2(x)}$, of a continuous uniform cylindrical tube to distributed random forces, $g(x, t)$, may be expressed by

$$\overline{y^2(x)} = \sum_r\sum_s \frac{\phi_r(x)\phi_s(x)}{16\pi^4 f_r^2 f_s^2}\int_0^{\infty} H_r^*(f)H_s(f)\int_0^{\ell}\int_0^{\ell} \phi_r(x)\,\phi_s(x)R(x,x',f)\,\mathrm{d}x\,\mathrm{d}x'\,\mathrm{d}f, \tag{22}$$

where $H_r^*$ and $H_s$ are complex complementary frequency response functions for the $r$th and $s$th modes, $R(x, x', f)$ is the spatial correlation density function and $x$ and $x'$ are points along the length of the tube. More detail is found in Pettigrew et al. (1978) and Axisa et al. (1990).

### 3.3. Periodic wake shedding

The response to periodic wake shedding must be calculated for any modes that are found to have Strouhal numbers in the range defined earlier. Assuming that the damping is small, peak vibration amplitudes, $Y(x)$, at resonance in the $i$th mode may be expressed as (Pettigrew et al., 1978)

$$Y(x) = \frac{\phi_i(x)}{8\pi^2 f_i^2\zeta_i}\int_0^{\ell} F_s(x')\phi_i(x')\,\mathrm{d}x' \tag{23}$$

where $F_s(x')$ is a distributed periodic force formulated by

$$F_s(x') = C_L\rho r(x)DU_p^2u^2(x)/2 \tag{24}$$

in which $C_L$ is the dynamic lift or drag coefficient defined earlier. Eq. (24) is a generalized form of Eq. (9).

Periodic wake shedding can result from very localized flow over a single span. Therefore, an effective velocity approach like that used for calculating fluidelastic instability may not be appropriate for periodic wake shedding, if the velocity distribution has large step changes in velocity. Regions with nozzles and inlets may have to be assessed separately.

### 3.4. Acoustic resonance

Susceptibility to acoustic resonance is not assessed by calculating the tube response. It is estimated by the method described earlier in this paper.

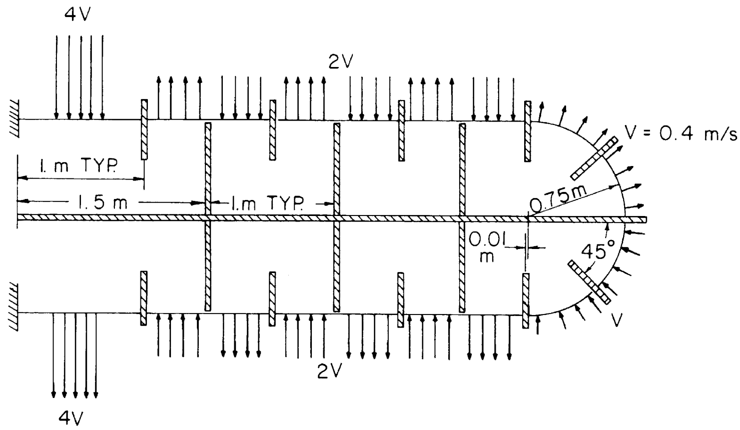


Fig. 7. Flow velocities, support locations and tube geometry for typical heat exchanger.

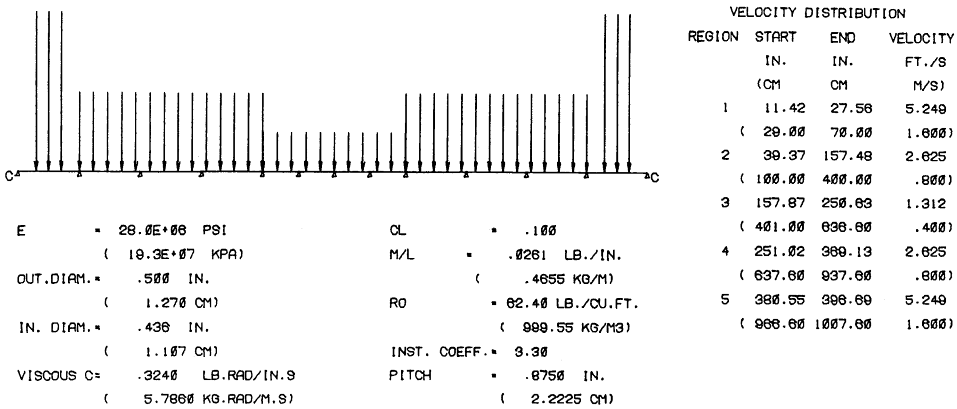


Fig. 8. Example of heat exchanger tube vibration analysis: input of vibration analysis code PIPO.

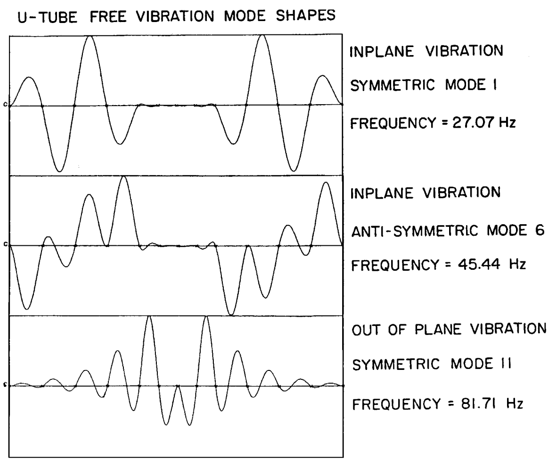


Fig. 9. Heat exchanger tube vibration: Typical free vibration analysis results.

### *3.5. Example of vibration analysis*

An example of a vibration analysis for a typical heat exchanger U-tube is illustrated in Figs. 7–9. This vibration analysis was done with the heat exchanger vibration analysis code PIPO1.

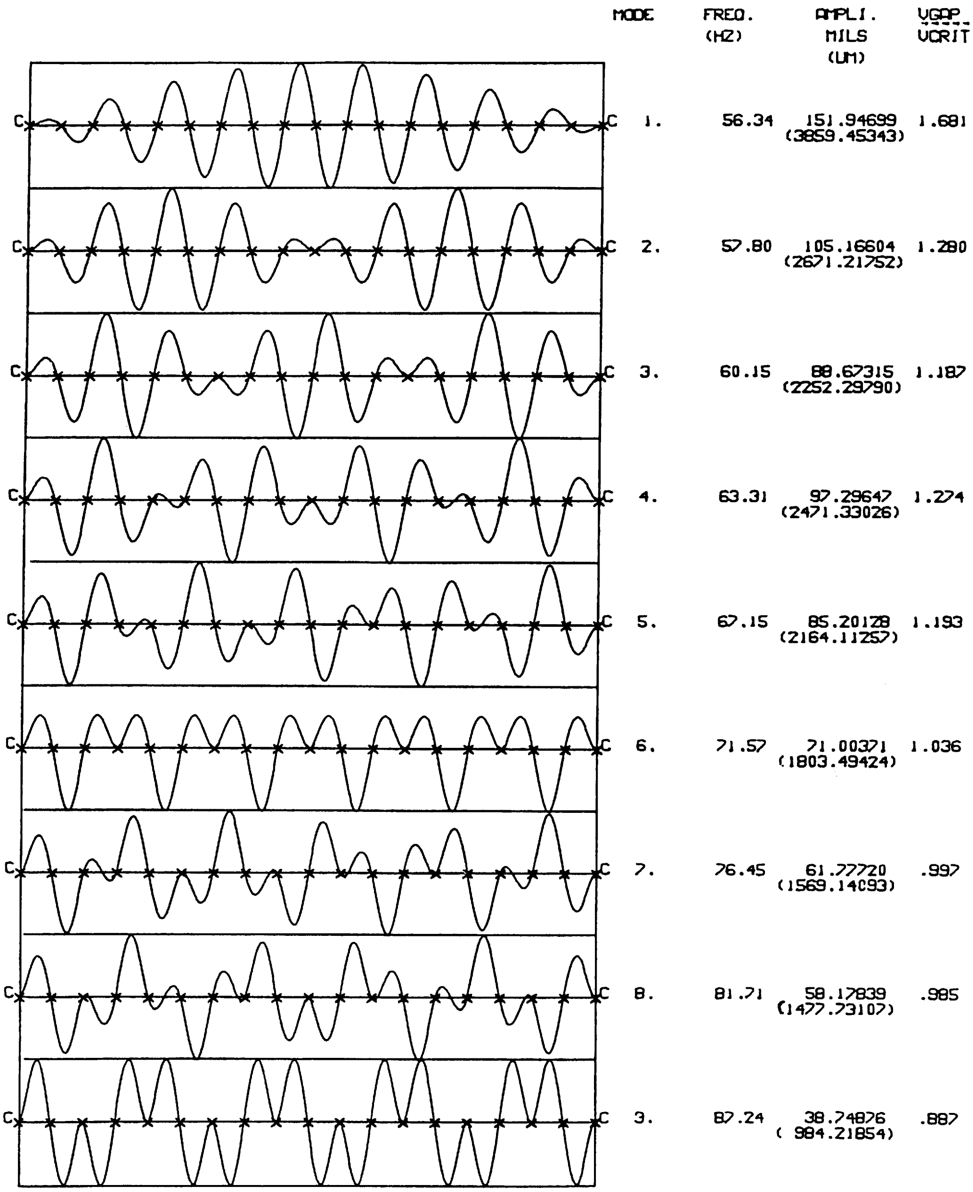


Fig. 10. Vibration mode shapes and vibration analysis results for typical condenser.

The tube support geometry and the support locations are shown in Fig. 7. The input data required for the PIPO1 code is outlined in Fig. 8. This figure shows the thermalhydraulic input in the form of pitch flow velocity distribution along the tube. Note the relatively high flow velocity in the inlet region near the tubesheet.

Typical free vibration analysis results are shown in Fig. 9 for selected vibration modes. The results include the vibration mode shapes and the natural frequencies. Fig. 10 shows the results of a fluidelastic instability analysis for the condenser tube described in Fig. 4 of (Pettigrew and Taylor, 2003). For some vibration modes, the ratio of actual to critical flow velocity for fluidelastic instability, $U_p/U_{pc}$, is greater than one. This means that fluidelastic instability is possible for this tube, which was subjected to abnormal flow conditions. In reality, fretting-wear and fatigue damage were observed in this condenser.

## 4. Fretting-wear damage considerations

Fretting-wear damage may be assessed using the following methodology. The total fretting-wear damage over the life of the heat exchanger must not exceed the allowable tube wall reduction or wear depth, $d_w$.

### 4.1. Fretting-wear assessment

Fretting-wear damage may be estimated using the following modified Archard equation:

$$\dot{V} = K_{FW}\dot{W}_N, \tag{25}$$

where $\dot{V}$ is the volume fretting-wear rate, $K_{FW}$ is the fretting-wear coefficient, and $\dot{W}_N$ is the normal work-rate. The work-rate is the available mechanical energy in the dynamic interaction between tube and support and is an appropriate parameter to predict fretting-wear damage. The work-rate may be calculated by performing a nonlinear time domain simulation of a multispan heat exchanger tube vibrating within its supports (Yetisir and Fisher, 1996). Alternately, the work-rate may be estimated with the following simplified expression (Yetisir et al., 1998, Pettigrew et al., 1999):

$$\dot{W}_N = 16\pi^3 f^3 m\ell\overline{y^2_{\max}}\zeta_s, \tag{26}$$

where $m$ is the total mass of tube per unit length, and $\left(\overline{y^2_{\max}}\right)$ and $f$ are, respectively, the maximum mean-square vibration amplitude and the natural frequency of the tube for the worst mode. The worst mode of a given region is defined as the mode of vibration that has the highest value for the normal work-rate term, $\dot{W}_N$, in Eq. (26). The length, $l$, is that of the span where the vibration amplitude is maximum and $\zeta_s$ is the damping ratio attributed to the supports.

### 4.2. Fretting-wear coefficients

Tube and tube-support materials should be chosen to minimize fretting-wear damage. Similar materials such as Inconel-600 (I600) tubes and I600 supports must be avoided. Incoloy-800 (I800), Inconel-690 (I690) and I600 tubes with 410S, 304L, 316L, 321SS or carbon steel supports are considered acceptable material combinations from a fretting-wear point of view. However, the choice of material is often governed by other considerations such as systems corrosion and heat transfer. The tube-supports, whether they be flat bars, lattice bars, broached holes, scalloped bars or circular holes must be deburred to make sure there are no sharp edges between tube and support. A fretting-wear coefficient, $K_{FW}$, of $20 \times 10^{-15}\,\mathrm{m^2/N}$ may be used as a first approximation for these tube and support material combinations (Guérout and Fisher, 1999). Other material combinations may be used. However, it would be desirable to obtain reliable fretting-wear coefficients before choosing the material combination.

### 4.3. Wear depth calculations

It may be assumed to be conservative that fretting-wear is taking place continuously for a total time, $T_s$, corresponding to the life of the component. The total fretting-wear volume, $V$, is calculated from Eq. (25):

$$V = T_s\dot{V} = T_sK_{FW}\dot{W}_N. \tag{27}$$

The resulting tube wall wear depth, $d_w$, can be calculated from the wear volume. This requires the relationship between $d_w$ and $V$. For example, for a tube within a circular hole or a scalloped bar, it may be assumed that the wear is taking place uniformly over the thickness, $L$, and half the circumference, $\pi D$, of the support. Thus,

$$d_W = 2V/(\pi DL). \tag{28a}$$

Alternatively, for lattice bars and for flat bars, FURs or AVBs, it may be assumed that wear is taking place on two sides and only on the tubes. Thus, the bars and the wear scars remain flat. This leads to

$$V = \frac{LD^2}{4}(2\theta - \sin 2\theta), \tag{28b}$$

where

$$\theta = \arccos\,[(D - 2d_w)/D]. \tag{28c}$$

Thus, the tube wall fretting-wear depth, $d_w$, shall be estimated using Eqs. (26), (27) and (28a) or (28b and c).

## 5. Acceptance criteria

The flow-induced vibration analysis should demonstrate that the steam generator or heat exchanger is acceptable by satisfying the design acceptance criteria outlined below.

### 5.1. Fluidelastic instability

The maximum flow velocity in a heat exchanger should be below the critical flow velocity for fluidelastic instability, $U_{pc}$, based on a fluidelastic instability constant, $K = 3.0$, for liquid flow, and as specified in (Pettigrew and Taylor, 2003) for two-phase flows. Thus

$$U_p/U_{pc} < 1.0. \tag{29}$$

A safety factor may be considered for added conservatism. For example, a ratio of 0.75 instead of 1.0 in Eq. (29) is sometimes specified in nuclear applications.

### 5.2. Random excitation

The vibration response to random excitation shall be sufficiently low to prevent excessive tube wall reduction due to fretting-wear. The tube wall fretting-wear depth, $d_w$, calculated as in the previous section for the entire life of the component should be less than a specified percentage of the nominal design tube wall thickness (e.g., 40%).

### 5.3. Periodic wake shedding

Resonance due to coincidence of tube frequency and periodic wake shedding frequency should be avoided. If the latter is not possible, the maximum r.m.s. tube vibration amplitude, $Y_{\max}$, at resonance should be less than 2% of the tube diameter, thus:

$$Y_{\max}\,\mathrm{r.m.s.} < 0.02D. \tag{30}$$

Below $0.02D$, the vibration amplitude is generally not sufficient to spatially correlate the formation of vortex shedding with the motion of the tube, thereby resulting in a much reduced uncorrelated vibration response.

### 5.4. Tube-to-support clearance

The tube-to-support clearance must be small enough to provide an effective support. Thus, pinned support conditions may be assumed provided the tube-to-support diametral clearance for drilled holes, broached holes, scallop bars, egg crates and lattice bars is equal to or less than 0.4 mm.

For flat bar-type U-bend supports, such as AVBs and FURs, the vibration analysis should satisfy the above criterion for random vibration for the in-plane direction while assuming that one (any one) support may not be effective in the U-bend region. In the out-of-plane direction, the diametral clearance between tube and flat bar support should be sufficiently small to provide effective support (e.g., $<0.1$ mm).

### 5.5. Acoustic resonance

The analysis should show that acoustic resonance conditions are avoided in the heat exchanger tube bundle. This should be based on at least two generally recognized criteria (e.g., Blevins and Bressler 1987; Ziada et al., 1989). The heat exchanger relevant parameter, whether it be shedding frequency, $f_s$, acoustic frequency, $f_{an}$, geometry, $P/D$, $L/D$ or resonance parameter, $G_s$, or $G_i$, should be at least 25% away from the resonance criteria or boundary, thus:

$$\text{Heat Exchanger Parameter} < 0.75 \text{ Acoustic Resonance Parameter.}$$

### 5.6. Two-phase flow regimes

Flow conditions leading to intermittent flow regime should be avoided in two-phase cross flow. This applies in particular to the U-bend region of nuclear steam generators.

## 6. Concluding remarks

This paper outlines design guidelines to avoid flow-induced vibration problems in shell-and-tube heat exchangers. The basic vibration excitation mechanisms and the required steps to carry out a flow-induced vibration analysis are reviewed. Flow considerations, dynamic parameters and fluidelastic instability were discussed in Part 1 of this paper. Part 2 covers forced vibration mechanisms, such as random turbulence excitation and vortex shedding, fretting-wear damage assessment and acceptance criteria.

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